314 Midterm prep question – Due Friday 18th November

1. Math Problems

i) Find dy/dx if $u = \sqrt{x}$ and $y = 1 - u$.

ii) Find dy/du if $u = \cos(2x-1)$, and $dy/dx = 2\sin(2x-1)$.

2. Order-of-magnitude estimates.

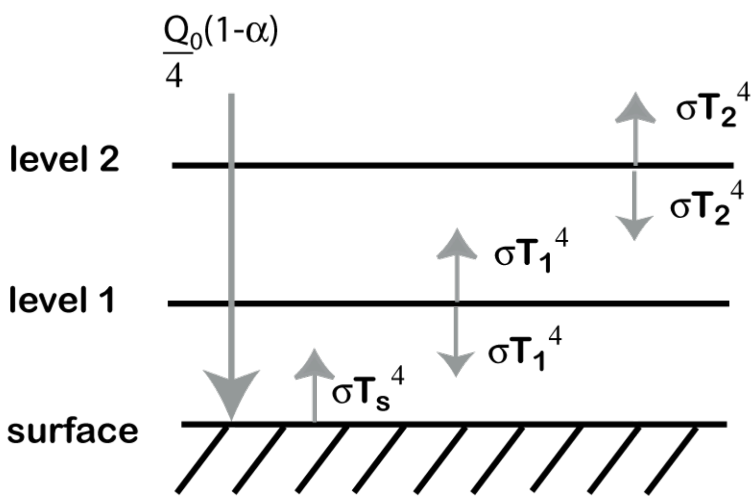
i) Are there more molecules of water in a teaspoon of water than there are teaspoons of water in all the world's oceans?

ii) How long would it be before the sun burned itself out if it consisted of burning coal (and an unseen and unexplained supply of oxygen!) and continued to radiate at its present output of 4×10^{26} J s^{-1}. The heat of combustion of coal is 3×10^{7} J kg^{-1}.

3. Space Physics

The magnetic field strength near the Van Allen radiation belts is about 10^{-5} T. Calculate the radii of the spiral paths of 0.1-MeV, 1-MeV, and 10-MeV protons in the van Allen belts. *(Numbers: 1 eV =1.6 x 10^{-19} Joules; 1 MeV = 10^{6} eV; mass of proton = 1.67 x 10^{-27} kg)*

4. Two-level greenhouse model



Schematic illustration of a two-layer atmosphere. The atmosphere is transparent to solar radiation, but each atmospheric layer perfectly absorbs the terrestrial radiation and reemits it both upwards and downwards.

Consider the climate model illustrated in the figure. In class we derived the temperature for a one-level atmosphere. This question asks you to expand it by considering an atmosphere comprised of two atmospheric layers. As before the atmosphere is assumed to be transparent to solar radiation, and perfectly absorbing in the infrared. So, for example, none of the infrared radiation emitted downwards from level 2 makes it to the surface – it is all absorbed by level 1, and reemitted. Equally none of the surface radiation makes it up to level 2 – it is absorbed by level 1 and reemitted.

i) Write down the energy balance that applies at the surface, at each of the two atmosphere levels, and the energy balance viewed from space (i.e., 4 equations). Remember that at each level there are sources and sinks of radiation, and at each level the sources of radiation must balance the sinks of radiation.

ii) By solving the simultaneous equations, show that the temperature of the surface is given by:

$$T_s = \left(\frac{3Q_0}{4\sigma}(1-\alpha)\right)^{\frac{1}{4}}.$$

5. Linearization question. How large are the errors involved?

i) We want to calculate $\sqrt{16.0002}$. Let $f(x) = x^{1/2}$. Using Taylor series show that

$$f(a+\varepsilon) \approx \sqrt{a} + \frac{\varepsilon}{2\sqrt{a}}$$

if ε is small. Hence calculate an approximate value for $\sqrt{16.0002}$.

ii) Now use a calculator to determine the percentage error from the real answer by having made this linear approximation.

6. Energy released in a thunderstorm.

Calculate the amount of latent energy released in a thunderstorm. Assume the thunderstorm cell is 10km to a side, and that about an inch of rain falls (2.5 cm).

Do a little research, and compare this energy to that released in an Hiroshima sized nuclear explosion.

Latent hear of condensation: 2270 kJ/kg (note units)

7. Diffusion and drag.

i) How long does it take for a spinning cup of coffee to slow down? Assume the dissipation of momentum is given by a diffusion equation:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial r^2}.$$

u is the velocity of the coffee, r is the distance from the center of the cup, and n is the kinematic viscosity of the coffee (0.3×10^{-6} m^2 s^{-1}). Use scale analysis and sensible estimates of the scale of a coffee cup, and the speed of well-stirred coffee.

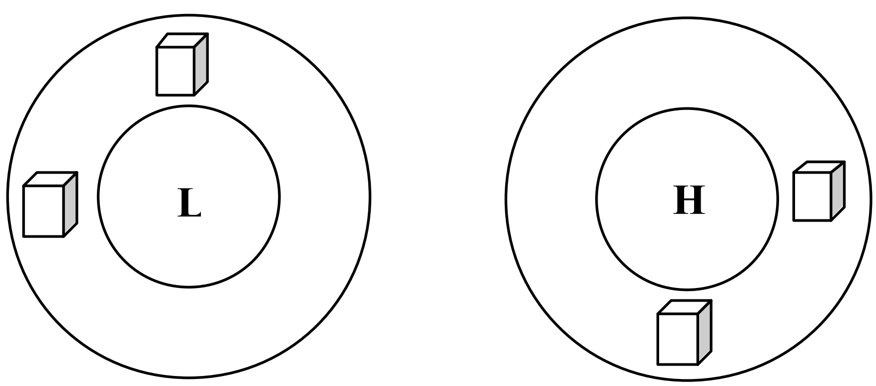
ii) Is your answer reasonable? What other factors might be involved other than diffusion of momentum.

8. Fluids and relative motion.

i) Mario skates south at 6.3 m/s. A fellow hockey player passes the puck to him. The puck moves at 10.4 m/s 30° west of south. What are the magnitude and direction of the puck's velocity, as observed by Mario?

ii) A snorkeler who can swim 1.00 m/s in still water aims her body directly across a 150-m-wide river. The current is 0.80 m/s. How far downstream does she end up on the opposite bank?

9. Geostrophic balance



Considering each diagram separately (reproduce these), draw and label arrows representing the pressure gradient force, the Coriolis force, surface friction, and the direction of motion for each parcel in the high and low-pressure systems in the *southern hemisphere*.