Why Does Vorticity Advection Increasing With Height Cause The Air To Go Up?

Consider vorticity equation:

$$\frac{\partial\zeta}{\partial t} = -\vec{V}\cdot\nabla_{p}(\zeta + f) - f\nabla\cdot\vec{V} \qquad (1)$$

Let $\eta = \zeta + F$, $\partial = \nabla\cdot\vec{V}$ and <u>assume steady state</u>

motion → recall $\frac{\partial(\_)}{\partial t} = -\vec{C}\cdot\nabla(\_)$ where $\vec{C} = C_{xd} + C_{y}$

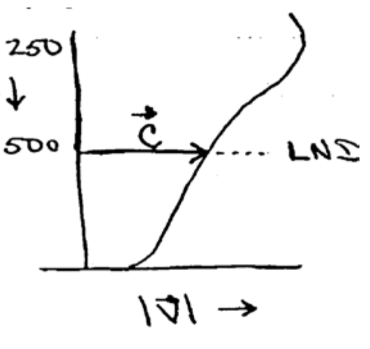
is the <u>speed of the trough or ridge</u>

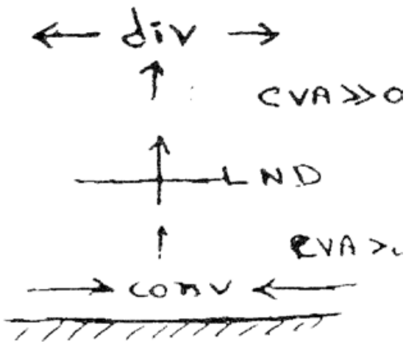
Rewrite (1) as

$$-\vec{C}\cdot\nabla\eta + \vec{V}\cdot\nabla_{p}(\zeta + f) = -f\zeta \qquad \text{or}$$

$$(\vec{V} - \vec{C})\cdot\nabla\eta = -f\zeta \qquad (2)$$

Now *assume vorticity advection increases with height* – this usually requires $\vec{V}$ to increase with height as shown. Note that the *level of non-divergence (LND) exists where* $\vec{C} = \vec{V}$. Assume W→E motion; consider x-component of (2) only: $(u - c_{x})\frac{\partial\eta}{\partial x} = -f\zeta$. *Now below LND,* $c_x > u$, $u - c_x < 0$ and since $\frac{\partial\eta}{\partial x} < 0$, LHS is > 0; $-f\zeta > 0$ means $\zeta < 0$ or *convergence below LND*. Similarly, above LND, $u > c_x \Rightarrow \zeta > 0$ or *divergence above* LND. By mass continuity, we must have *upward motion in this region.*





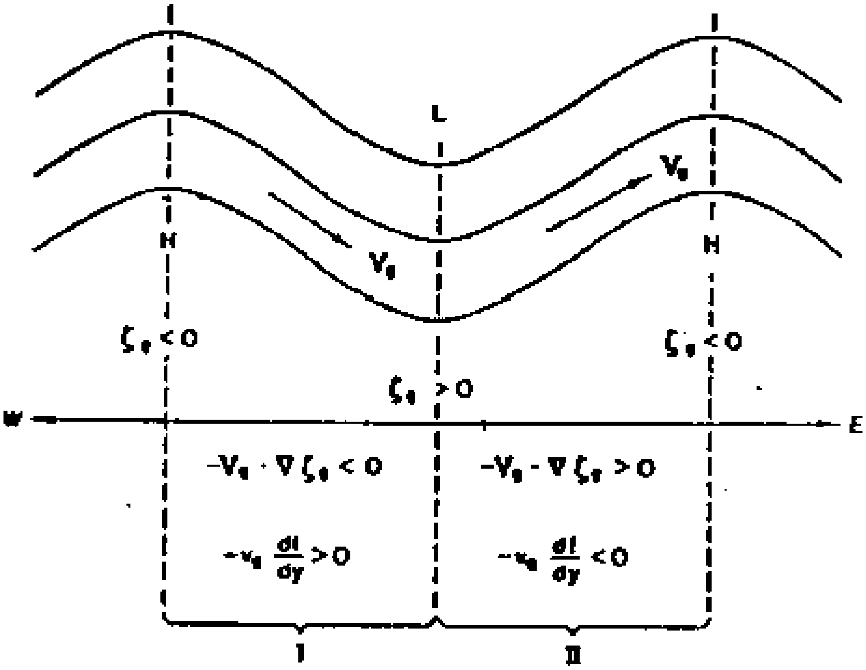


Fig. 6.7 Schematic 500 mb geopotential field showing regions of positive and negative advections of relative and planetary vorticity.

6.2 DEVELOPMENT OF THE QUASI-GEOSTROPHIC SYSTEM

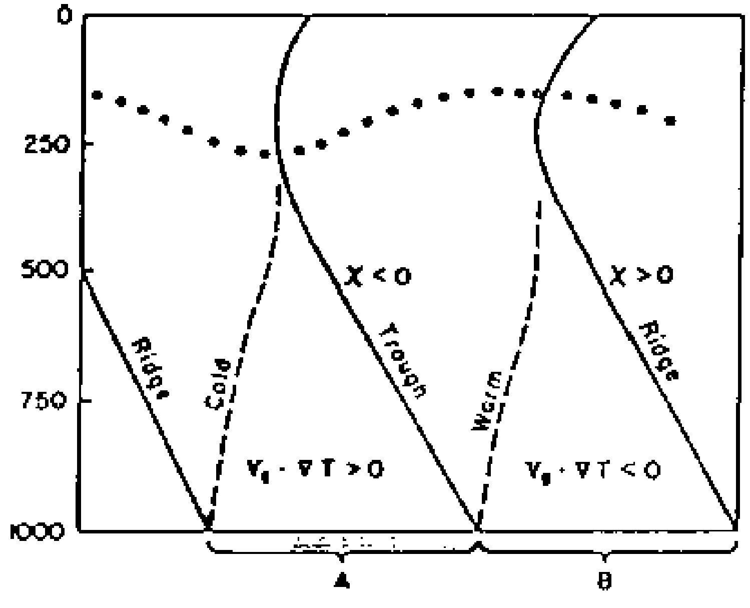


Fig. 6.8 East-west section through a developing synoptic disturbance showing the relationship of temperature advection to the upper level height tendencies. A and B designate, respectively, regions of cold advection and warm advection in the lower troposphere.

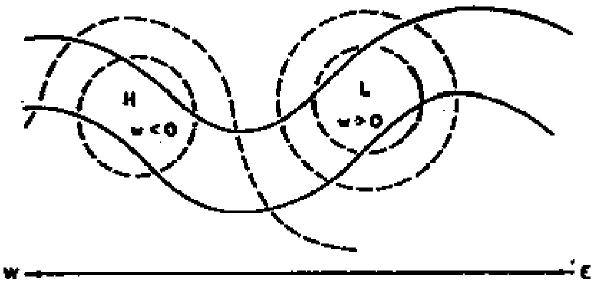


Fig. 6.9 Schematic 500 mb contours (solid lines) and 1000 mb contours (dashed lines) indicating regions of strong motion due to differential vorticity advection.

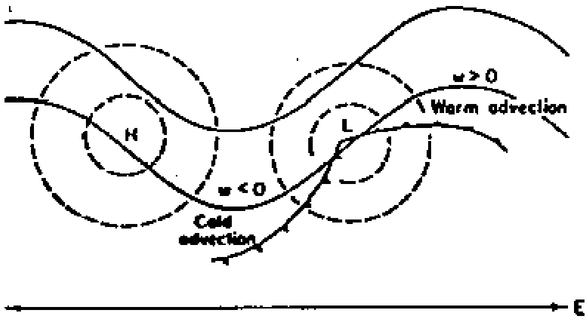


Fig. 6.10 Schematic 500 mb contours (thin solid lines), 1000 mb contours (dashed lines), and surface fronts (heavy lines) indicating regions of strong vertical motion due to temperature advection.

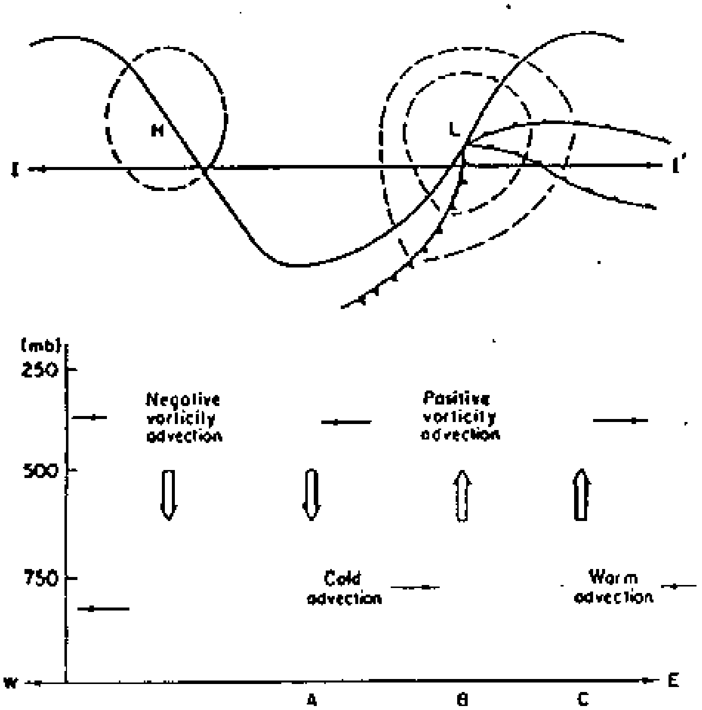


Fig. 6.11 Secondary circulation associated with a developing baroclinic wave: (top) schematic 500 mb contour (solid line), 1000 mb contours (dashed lines), and surface fronts; (bottom) vertical profile through the line II indicating the divergent and vertical motion fields.

Western Region Technical Attachment
July 19, 1966
No. 20-66

The Physical Relationship Between Vorticity Advection and Vertical Motion

An occasional review of the physics underlying the use of routine analyses and/or forecasting procedures is a good practice to follow. It emphasizes the proper use of a technique by bringing back to each forecaster's attention the strong and weak points of the technique, plus any restrictions regarding its use. This Technical Attachment reviews the relationship between positive vorticity advection at 500 mb as indicated on facsimile charts, and vertical motion.

Discussion of vorticity may seem to be out of date in the light of comments in previous Attachments. These have suggested that the use of vorticity charts has reached its zenith and will probably be on the decline in the future as NMC improves primitive equation model NWP forecasts which do not involve vorticity. While this is still considered true, vorticity charts based on the barotropic model are expected to be with us for several years to come.

The vorticity lines on the current facsimile 500 mb analyses and prognoses are isopleths of absolute vorticity – i.e., relative vorticity plus the Coriolis parameter – and are labeled in units of 10^{-5} per second. Vorticity isopleths have two important uses in operational forecasting:

1) They add detail to the 500 mb contours by clearly locating short-wave troughs and ridges when the associated curvature of the contours is small or missing (see Figure 1).
2) They make available a useful tool -- indicated vorticity advection – for forecasting clouds and precipitaion.

With regard to the latter, the use of vorticity advection is an intermediate step taken to indicate the sign of vertical motion. If we were given the vertical motion directly (and we expect that P.E. forecast vertical-motion charts will soon be transmitted on facsimile) there would be no need to use the vorticity advection as a vertical-motion indicator.

The relationship between positive vorticity advection at 500 mb and upward vertical motion is not unique. One of the best ways of seeing this is to examine the simplified vorticity equation (1):

$$(V - C) \cdot \nabla\eta = KD_{iv_p}V$$

V is the wind at a given point on the constant-pressure surface, C is the translation speed of the synoptic system, e.g., trough or ridge movement, and $\nabla\eta$ is the gradient of the vorticity isopleths at the given point under consideration on the constant-pressure surface. K is a constant for purposes of this discussion.

$V\cdot\nabla\eta$ is the vector notation for vorticity advection; $C\cdot\nabla\eta$ is the local change of vorticity at a point due to the movement of troughs and/or ridges, assuming no change of shape and intensity of the system as it moves. What this equation says is that the indicated vorticity advection on a constant-pressure surface and the local change indicated by the speed of the trough or ridge associated with this advection determines whether divergence or convergence is taking place in the indicated vorticity advection area.

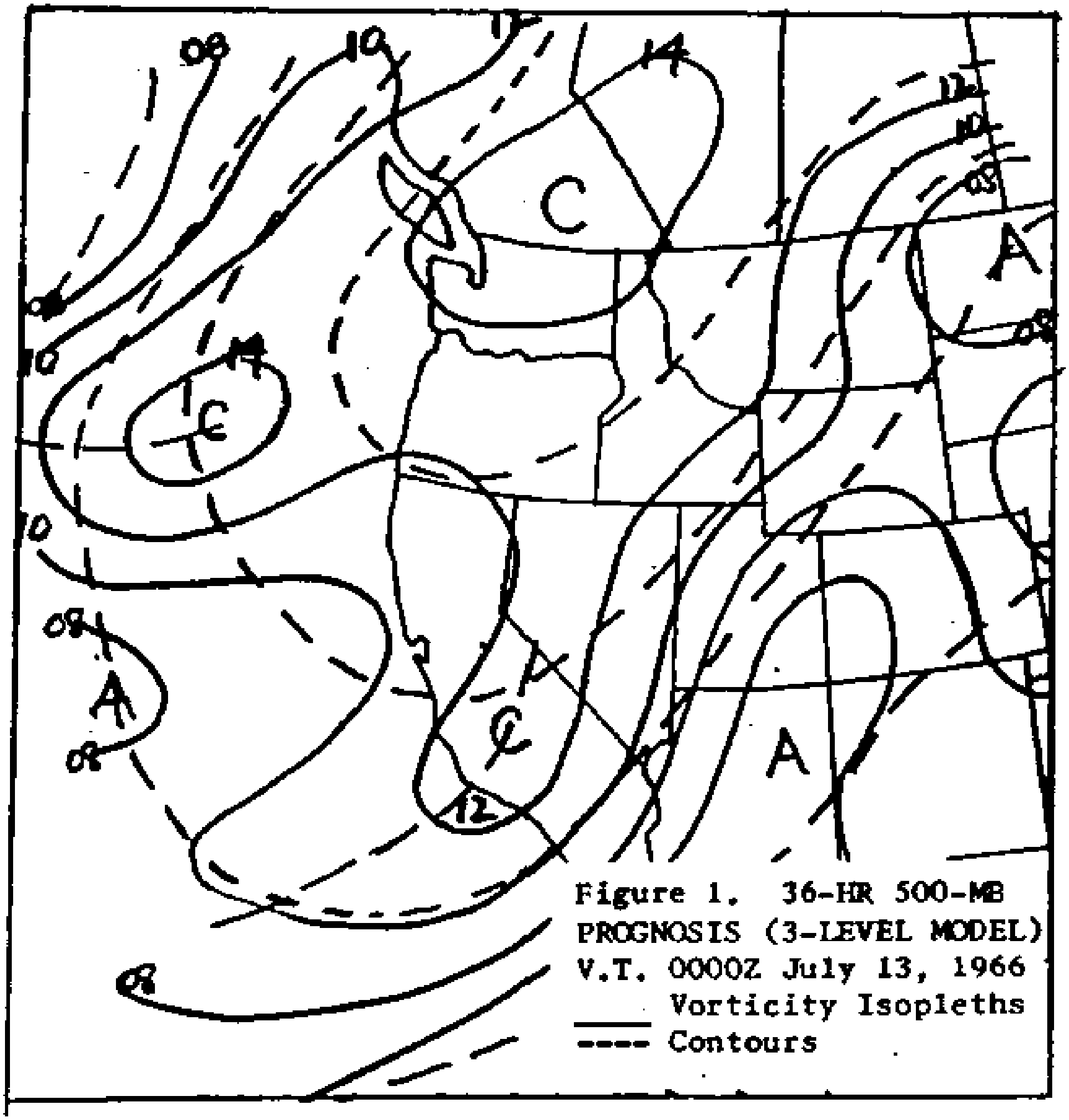


Figure 1. 36-HR 500-MB PROGNOSIS (3-LEVEL MODEL) V.T. 0000Z July 13, 1966

____ Vorticity Isopleths

---- Contours

If we assume, as is usually done, that the level of nondivergence is close to the 500 mb surface, then $(V-C)\cdot\nabla\eta = 0$ or, saying this in another way, the vorticity advection determines the motion of the associated trough or ridge, i.e., $V\cdot\nabla\eta = C\cdot\nabla\eta$. But this doesn't tell us anything about the sign of the vertical motion. Therefore, positive vorticity advection can be associated just as well with upward as downward vertical motion. It is only when we assume that the troughs and ridges have very little slope (i.e., the vorticity gradient, $\nabla\eta$, doesn't change sign with height) and that the wind component perpendicular to the vorticity lines increases with height that we can use positive vorticity advection at 500 mb as an indicator of upward vertical motion. To demonstrate this, let us apply the vorticity equation separately at 850, 500, and 300 mb and assume the wind increases with height, with no large (> 10 degrees) changes in wind direction.

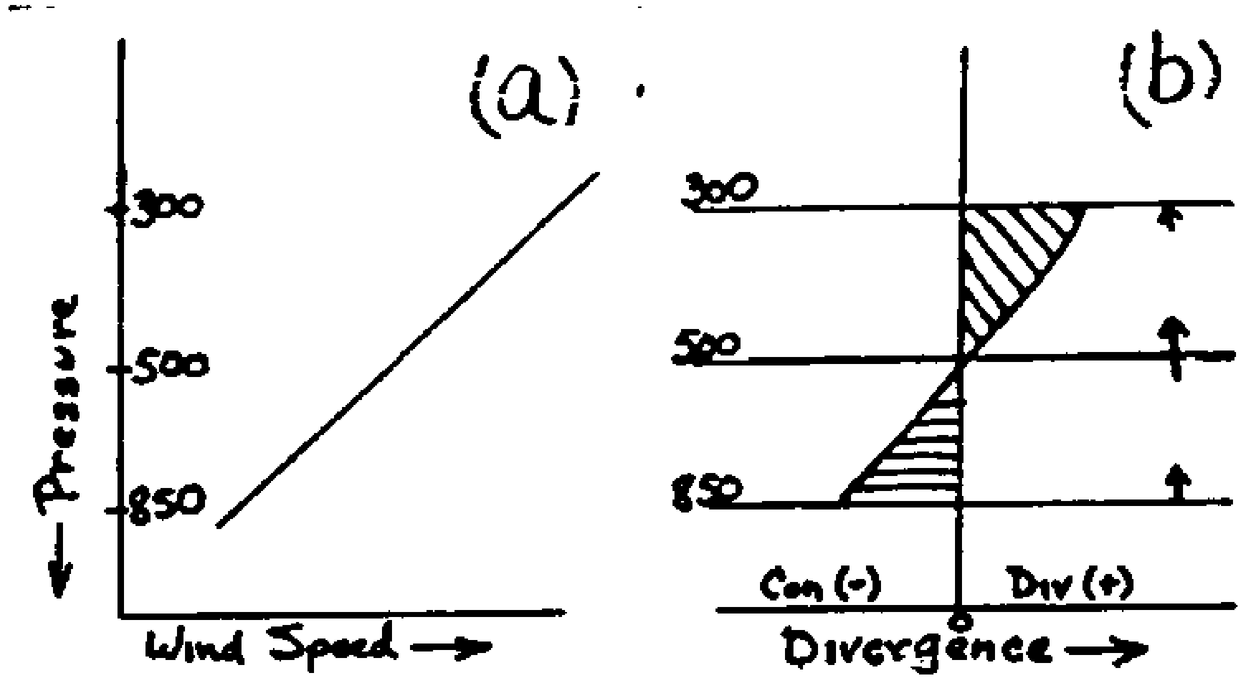


Figure 2. Schematic drawings of vertical wind shear and vertical distribution of divergence.

Assuming vorticity values decrease downstream, as is the case ahead of a trough:

At 850 mb $(V<C)$ then in the equation $-K = -\eta$

$$(V-C)\cdot\nabla\eta = -K\, D_{ivp}V$$

the term $(V-C)$ is negative and this requires that $D_{ivp}V$ be negative: i.e., convergence.

At 500 mb $(V = C)$ then $D_{ivp}V = 0$

At 300 mb $(V > C)$ then $(V-C)$ is positive, which indicates divergence. This is the case when wind blows through the contour pattern.

These results are plotted schematically in Figure 2b, assuming a linear change of divergence with height.

Thus, ahead of a trough, convergence is indicated below 500 mb and divergence above, and this calls for upward motion with a maximum at 500 mb. This is the model we assume when we relate positive vorticity advection at 500 mb to upward vertical motion and/or to convergent-type synoptic patterns at or near the surface.

Note that if there is little or no vertical wind shear, positive vorticity advection at 500 mb does not indicate large-scale upward motion. Also, there are occasions when troughs move faster that the indicated vorticity advection. Usually they are large amplitude troughs associated with old occluded systems. When this occurs, downward motion is associated with positve vorticity advection and upward motion with negative vorticity advection, i.e., upward motion behind the trough instead of ahead of it. French and Johannessen, in their article on the association of indicated 300 mb positve vorticity advection and cirrus clouds (2), found this reverse situation to occur 14% of the time in the data they studied. Figure 3 is an example of this situation taken from their article. Pilot reports of overcast cirrus (i.e., assumed upward vertical motion) along the flight path are indicated by the black stripe and broken cirrus by the hatched stripe in the figure.

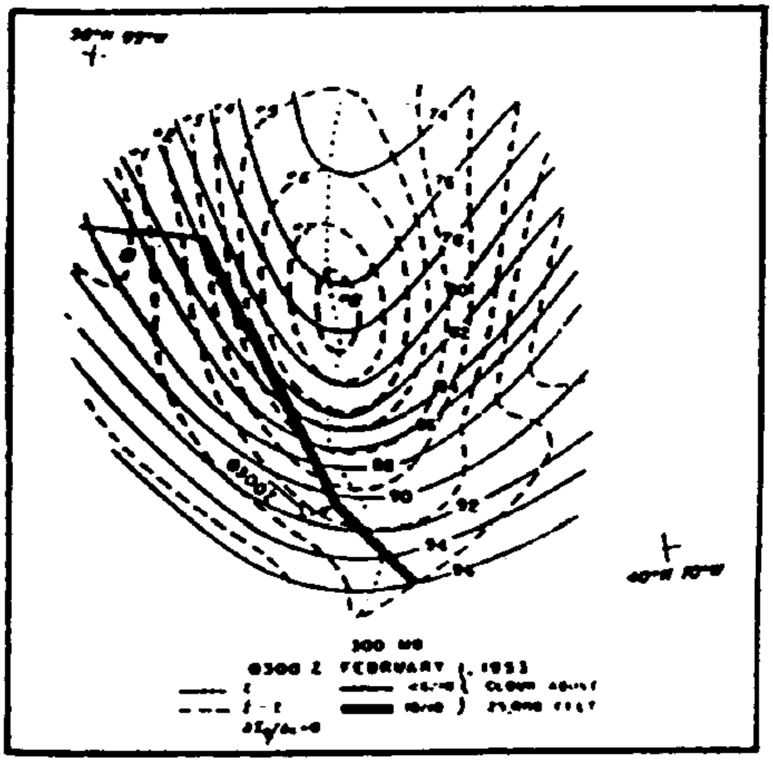


Figure 3. 300 mb chart 0300Z February 1, 1953. Dashed lines are isopleths of (Z-Z) at 100-foot intervals; these can be considered isopleths of relative vorticity. Solid lines are contours at 200-foot intervals. The track of the observing aircraft is indicated and its position at 0300Z located by the arrow.

Other limitations in using positive vorticity advection as an indicator of upward vertical motion are: 1) errors in the barotropic forecast; 2) vertical motions related to temperature advection below 500 mb (the theoretical relationship which Sutcliffe pointed out in 1947 (3). A good discussion of this second point and others is given in the NAWAC Manual (4).

The relationship between clouds and precipitation and upward vertical motion is not simple. The amount of moisture available is an important item in this relationship; also, the magnitude of the vertical motion is involved. In general, positive vorticity advection areas are usually much larger than the related cloud shield. In our Region, forecasters have found empirically that when positive vorticity advection areas are associated with precipitation, the absolute value of the vorticity usually exceeds 12×10^{-5} per second.

In summary, the simple relationship between positive vorticity advection and cloudiness, while very useful, has important limitations. Therefore, this relationship is only one of several indicators that the forecaster must consider in preparing cloud and precipitation forecasts. For example, moisture distribution, temperature advection, air-mass stability, and surface heating or orographic lifting must also be taken into account.

References:

(1) Cressman, G.P., "An Approximate Method of Divergence Measurement", Journal of Meteorology, April 1954, Pages 83-84.

(2) French, J. E., and Johannessen, K. R., "Forecasting High Clouds from High-Level Constant-Pressure Charts", Proceedings of the Toronto Meteorological Conference 1953, Pages 160-171.

(3) Sutcliffe, R. C., "A Contribution to the Problem of Development", Quarterly Journal of Royal Meteorological Society, October 1957, Pages 370-383.

(4) NAWAC Manual, Part II, November 1961, Pages 9-14.

Thermal Vorticity Interpretation

$$\zeta_T = \zeta_u - \zeta_h = \frac{g}{fo}\nabla^2(Z_u - Z_L) = \frac{g}{f_o}\nabla^2 h$$

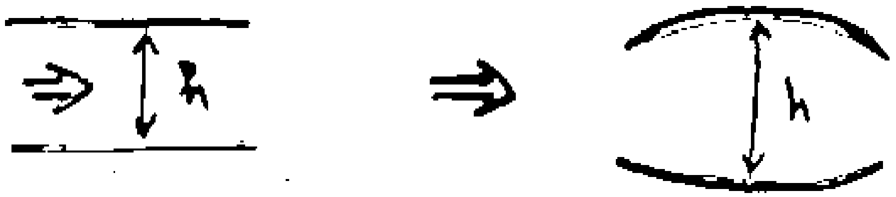
1. Vorticity advection increasing with height:

$$\frac{\partial}{\partial t}(\zeta_u) > \frac{\partial}{\partial t}(\zeta_L) \Rightarrow \frac{\partial}{\partial t}(\zeta_u - \zeta_L) > 0 \Rightarrow \frac{\partial}{\partial t}\left[\nabla^2(Z_u - Z_L)\right] > 0 \Rightarrow \frac{\partial}{\partial t}\left(\nabla^2 h\right) > 0 \Rightarrow \frac{\partial h}{\partial t} < 0$$

Thickness decreases if neglect T.A., only $\omega < 0$ (↑) can keep Thickness change hydrostatic.

2. Temperature advection:

W.A.



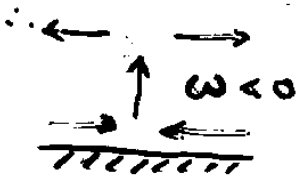
ζ_g decrease

ζ_g increase

$$\frac{\partial h}{\partial t} > 0 \Rightarrow \frac{\partial}{\partial t}\zeta_T < 0 \Rightarrow$$

ζ_u decrease
↓
Div. aloft

ζ_L increase
↓
conv. Below



∴ warm advection → rising motion