Adaptive Mesh Refinement for Numerical Tsunami Modeling

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ABSTRACT

Tsunamis belong to a class of geophysical problems with vastly different spatial scales of interest. For instance, the wave propagation of a global scale tsunami may result in wave run-up and inundation that varies greatly even along local stretches of coastline. This diversity of scales presents a difficulty to numerical modelers—the accuracy that is desired at a local scale requires a grid resolution that is simply not feasible to use at the global scale. Additionally, waves that propagate throughout the global domain may be concentrated in localized areas at a given time. Therefore, using fixed telescoping grids is not always a satisfactory solution.

We have used adaptive mesh refinement algorithms, originally developed for gas dynamics, to numerically model global scale tsunamis. These algorithms allow regions with grids of varying refinement where the solution has steep gradients or other features of interest. The regions of refinement may move adaptively with the solution over time, allowing various resolutions in a single global scale computation.

The Shallow Water Equations with Topography

- ▷ Equations for Depth and Momentum

$$\begin{bmatrix} h \\ hu \\ hv \end{bmatrix}_t + \begin{bmatrix} hu \\ hu^2+\frac{1}{2}gh^2 \\ huv \end{bmatrix}_x + \begin{bmatrix} hv \\ huv \\ hv^2+\frac{1}{2}gh^2 \end{bmatrix}_y = \begin{bmatrix} 0 \\ -ghb_x \\ -ghb_y \end{bmatrix}$$

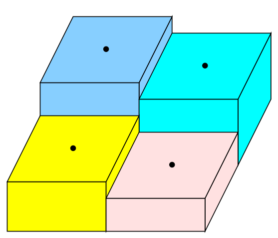
- ▷ These Equations are a Hyperbolic Conservation Law

$$\partial_t q + \partial_x f(q,\vec{x},t) + \partial_y g(q,\vec{x},t) = \psi$$

The Wave Propagation Method

$$\partial_t q + \partial_x f(q,\vec{x},t) + \partial_y g(q,\vec{x},t) = \psi$$

$$Q^n_{ij} \approx \frac{1}{\Delta x \Delta y} \iint_{C_{ij}} q(x,y,t_n)\,dx\,dy$$



The Numerical Integrator on Each Grid:

- ▷ Based on a wave-propagation method developed for hyperbolic conservation laws.
- ▷ Finite volume discretization approximates a discrete integral conservation law.
- ▷ Grid-cell values, Q^n_{ij}, represent average conserved quantities in each cell.
- ▷ Update in each timestep comes from solving 1D normal Riemann problems.

Properties of the Method:

- ▷ Numerically conservative.
- ▷ Second-order accurate for smooth solutions.
- ▷ Shock-capturing allows convergence to propagating bores.
- ▷ Dry-regions are captured in the computing domain.

Adaptive Mesh Refinement

- ▷ A single coarse Cartesian grid serves as the parent grid.
- ▷ Different scales are accommodated by multiple Cartesian sub-grids of different resolutions.
- ▷ Refinement regions evolve in time by adaptively generating new grids and averaging old grids, based on refinement algorithms.
- ▷ Propagating waves can be highly resolved by refined grids that move with the waves.
- ▷ Regions of interest can be highly resolved as waves arrive.
- ▷ Computation is not wasted in nearly static regions, since such regions can be accurately modelled by very coarse grids.
- ▷ One large computing domain reduces difficulties associated with computational boundaries.

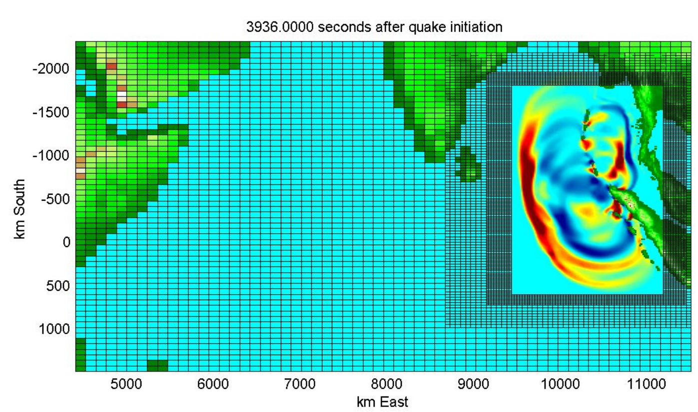
Shoreline Capturing

- ▷ Dry land is also part of the computing domain, eliminating the need for special treatment of the shorelines.
- ▷ In a dry region the finite volume cells simply have zero depth.
- ▷ Inundation at shorelines is naturally captured by allowing grid cells to fill up with water or drain out.
- ▷ We have developed Riemann solvers for this application that allow cells to dry without generating negative depths.
- ▷ It has been our goal to design Riemann solvers that accurately capture wave run-up onto dry land, without excessive computational cost.
- ▷ The conservative form of the shallow water equations, and the wave propagation algorithm, allow convergence to bores or waves at the breaking point.

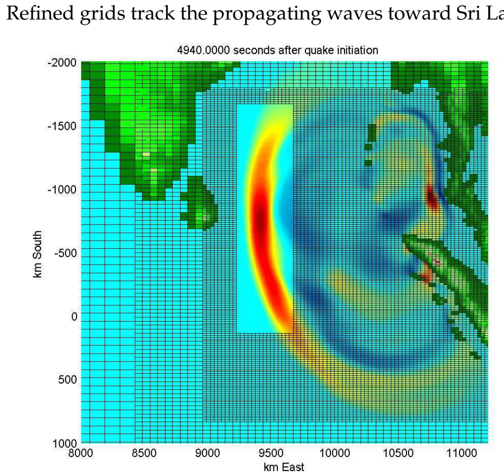
Grid Refinement on the Indian Ocean

Example: The Indian Ocean with five levels of refinement

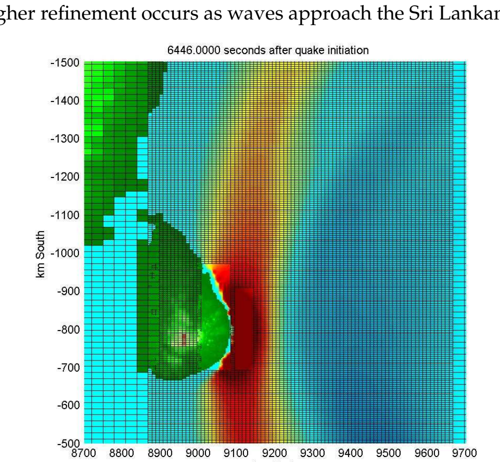
Grid lines on the finest grids in each figure are omitted for clarity



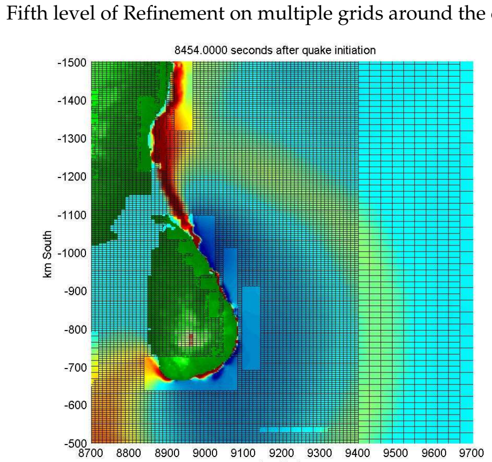
Refined grids track the propagating waves toward Sri Lanka



Higher refinement occurs as waves approach the Sri Lankan coast

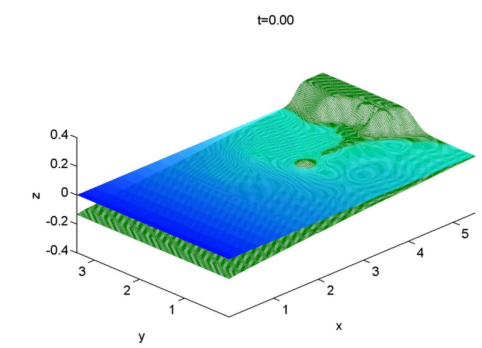


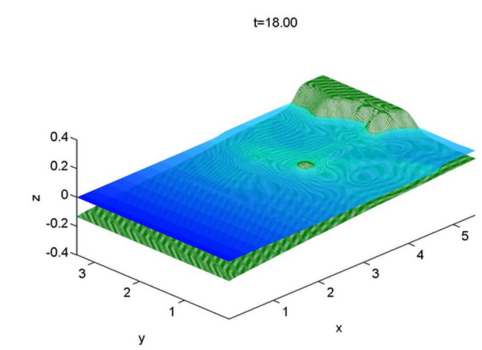
Fifth level of Refinement on multiple grids around the coast

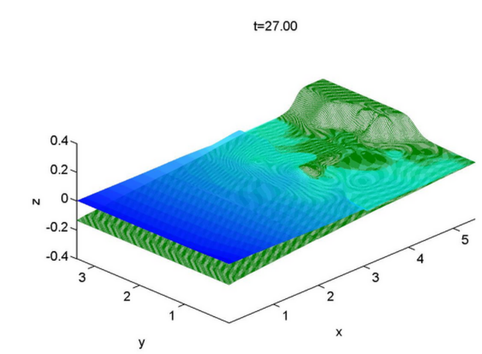


Inundation at a Local Scale

Snapshots of a problem from the Long-Wave Workshop 2004 [6]







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[6] Yeh et. al. 3rd International Workshop on Long-wave Runup Models. Results available at http://www.cee.cornell.edu/longwave.